Dealing with Data Gradients: “Backing Out” & Calibration

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CMPT 858

Sources for Parameter Estimates

- Surveillance data
- Controlled trials
- Outbreak data
- Clinical reports data
- Intervention outcomes studies
- Calibration to historic data
- Expert judgement
- Systematic reviews

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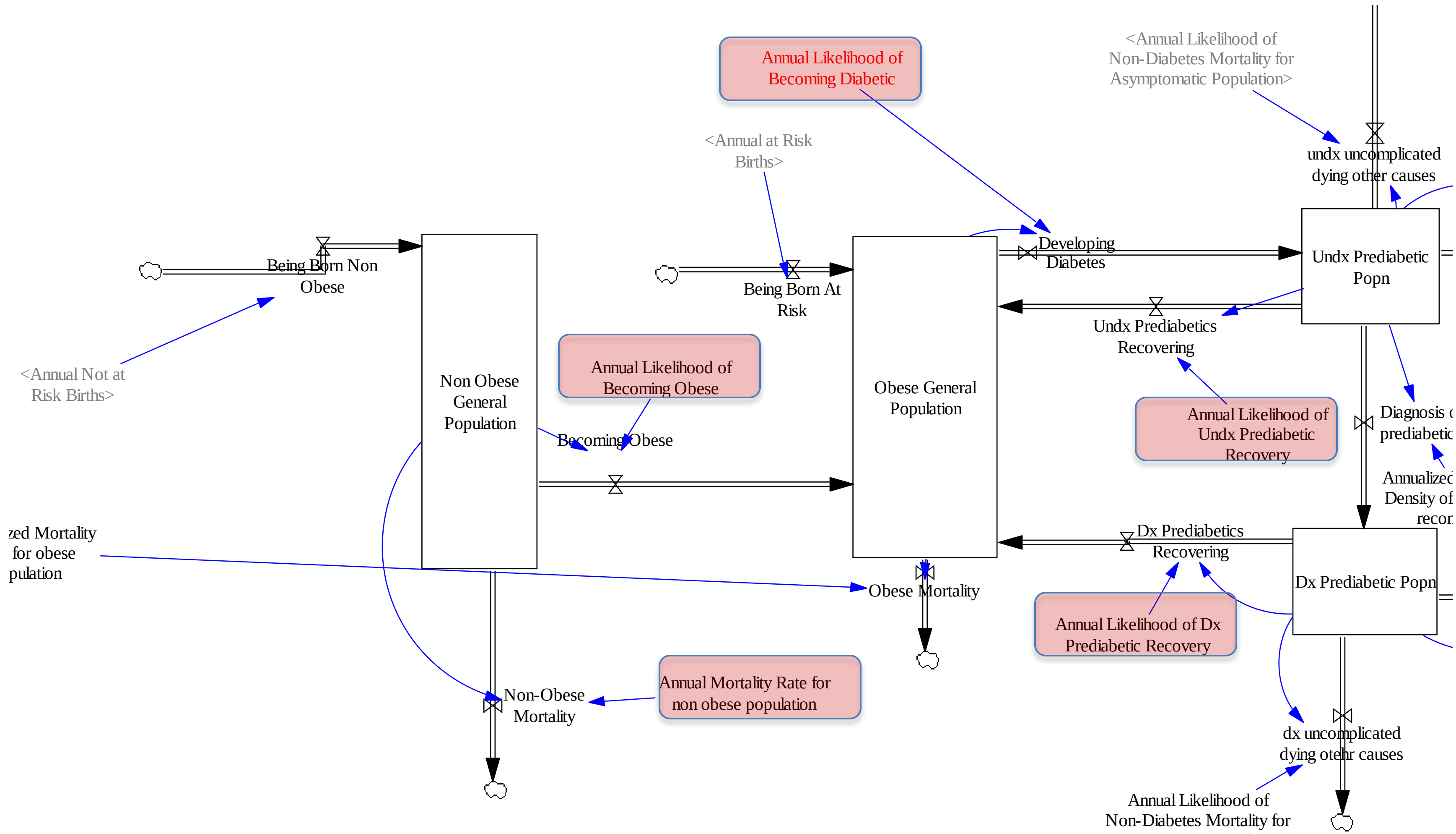
Introduction of Parameter Estimates



Sensitivity Analyses

- Same relative or absolute uncertainty in different parameters may have hugely different effect on outcomes or decisions
- Help identify parameters that strongly affect
  - Key model results
  - Choice between policies
- We place more emphasis in parameter estimation into parameters exhibiting high sensitivity

Dealing with Data Gradients

- Often we don’t have reliable information on *some* parameters, but do have other data
  - Some parameters may not be observable, but some closely related observable data is available
  - Sometimes the data doesn’t have the detailed breakdown needed to specifically address one parameter
    - Available data could specify sum of a bunch of flows or stocks
    - Available data could specify some function of several quantities in the model (e.g. prevalence)
- Some parameters may implicitly capture a large set of factors not explicitly represented in model
- There are two big ways of dealing with this: manually “backing out”, and automated calibration

“Backing Out”

- Sometimes we can manually take several aggregate pieces of data, and use them to collectively figure out what more detailed data might be
- Frequently this process involves imposing some (sometimes quite strong) assumptions
  - Combining data from different epidemiological contexts (national data used for provincial study)
  - Equilibrium assumptions (e.g. assumes stock is in equilibrium. Cf deriving prevalence from incidence)
  - Independence of factors (e.g. two different risk factors convey independent risks)

Example

- Suppose we seek to find out the sex-specific prevalence of diabetes in some population
- Suppose we know from published sources
  - The breakdown of the population by sex (c_M, c_F)
  - The population-wide prevalence of diabetes (p_T)
  - The prevalence rate ratio of diabetes in women when compared to men (rr_F)
- We can “back out” the sex-specific prevalence from these aggregate data (p_F, p_M)
- Here we can do this “backing out” without imposing assumptions

Backing Out

\# male diabetics + # female diabetics = # diabetics

$(p_M * c_M) \quad + \quad (p_F * c_F) \quad = p_T * (c_M + c_F)$

- Further, we know that $p_F / p_M = rr_F \Rightarrow p_F = p_M * rr_F$
- Thus

$(p_M * c_M) \quad + \quad ((p_M * rr_F) * c_F) \quad = p_T * (c_M + c_F)$

$p_M * (c_M + rr_F * c_F) = p_T * (c_M + c_F)$

- Thus
  - $p_M = p_T * (c_M + c_F) / (c_M + rr_F * c_F)$
  - $p_F = p_M * rr_F = rr_F * p_T * (c_M + c_F) / (c_M + rr_F * c_F)$

Disadvantages of “Backing Out”

- Backing out often involves questionable assumptions (independence, equilibrium, etc.)
- Sometimes a model is complex, with several related known pieces
  – Even thought we may know a lot of pieces of information, it would be extremely complex (or involve too many assumptions) to try to back out several pieces simultaneously

Another Example: Joint & Marginal Prevalence

	Rural	**Urban**	
Male	p_{MR}	p_{MU}	p_M
Female	p_{FR}	p_{MU}	p_F
	p_R	p_U	

Perhaps we know

- The count of people in each { Sex, Geographic } category
- The marginal prevalences (p_R, p_U , p_M , p_F)

We need at least one more constraint

- One possibility: assume $p_{MR} / p_{MU} = p_R / p_U$

We can then derive the prevalences in each { Sex, Geographic } category

Calibration: “Triangulating” from Diverse Data Sources

- Calibration involves “tuning” values of less well known parameters to best match observed data
  - Often try to match against *many* time series or pieces of data at once
  - Idea is trying to get the software to answer the question: “What must these (less known) parameters be in order to explain all these different sources of data I see”
- Sometimes we learn from this that our model structure just can’t produce the patterns!

Calibration

- Calibration helps us find a reasonable (specifics for) "dynamic hypothesis" that explains the observed data
  - Not necessarily the truth, but probably a reasonably good guess – at the least, a consistent guess
- Calibration helps us leverage the large amounts of diffuse information we may have at our disposal, but which cannot be used to directly parameterize the model
- Calibration helps us falsify models

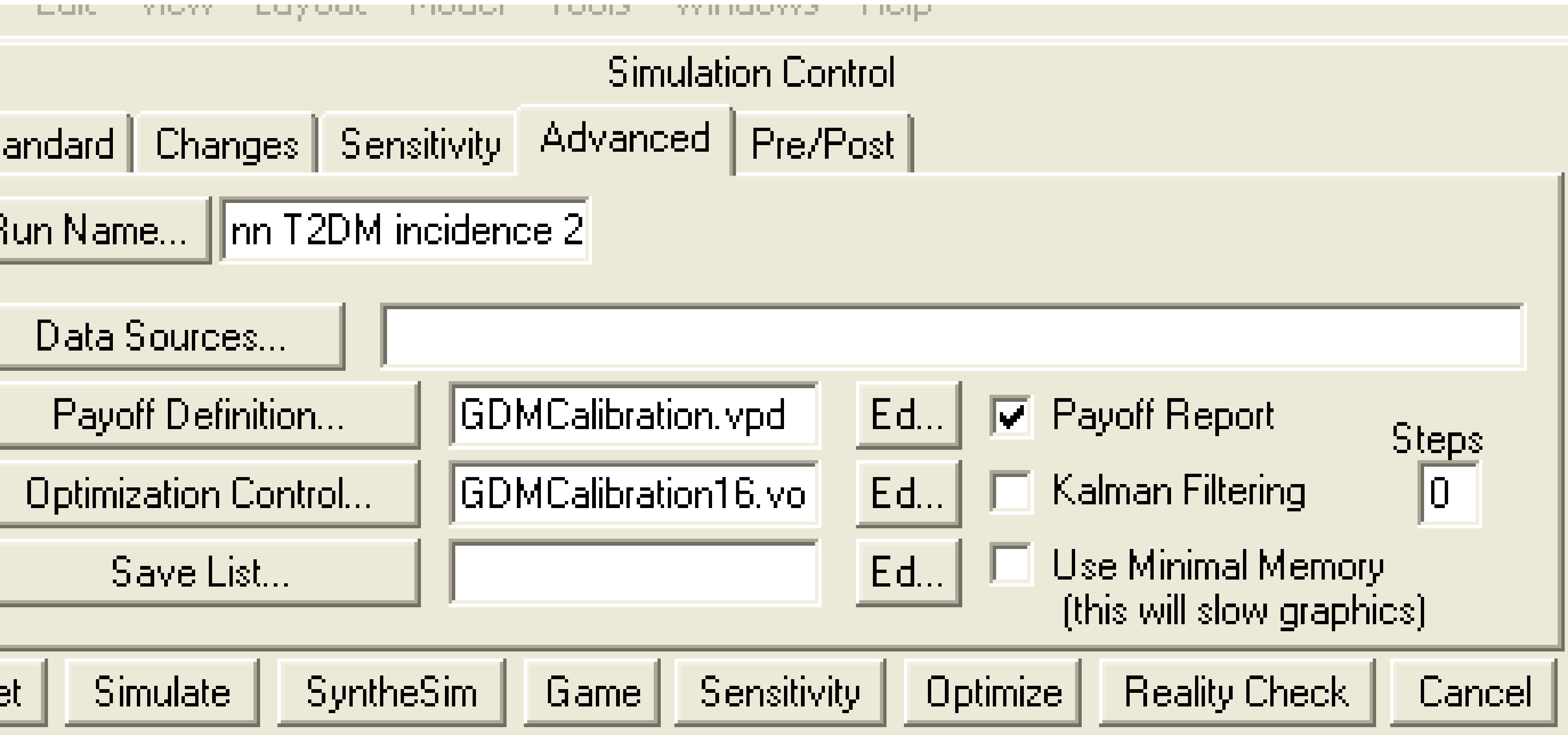
Calibration: A Bit of the How

- Calibration uses a (global) optimization algorithm to try to adjust unknown parameters so that it automatically matches an arbitrarily large set of data
- The data (often in the form of time series) forms constraints on the calibration
- The optimization algorithm will run the model many (minimally, thousands, typically 100K or more) times to find the “best” match for all of the data

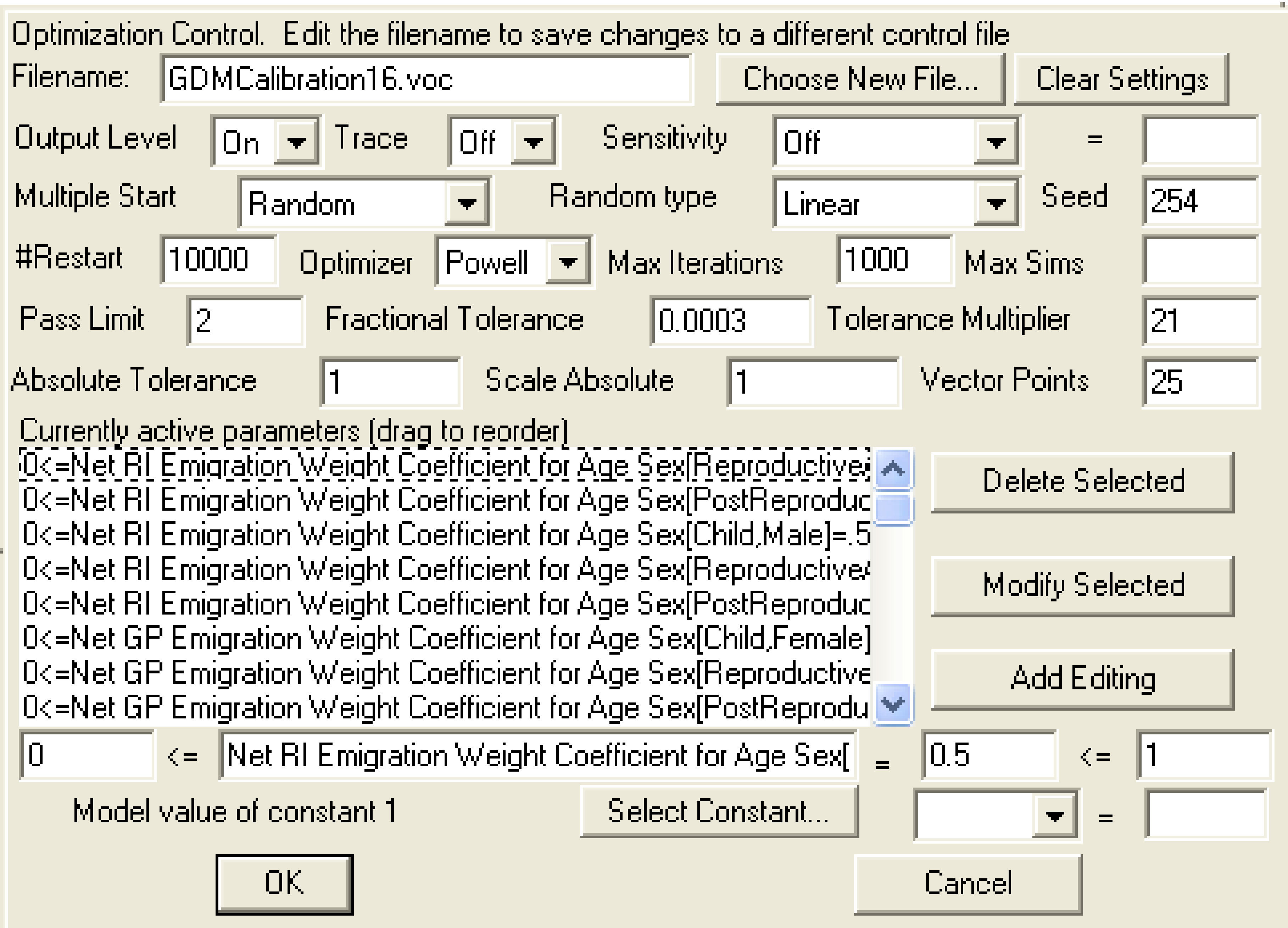
Example Global Optimization Algorithm

- Starts at random position, tries to improve match (minimize error) by
  - “Tweaking” parameters
  - Running Model
  - Recording error function
- Keeps on improving until reaches “local minimum” in error of fit
  - May add some randomness to knock out of local minima

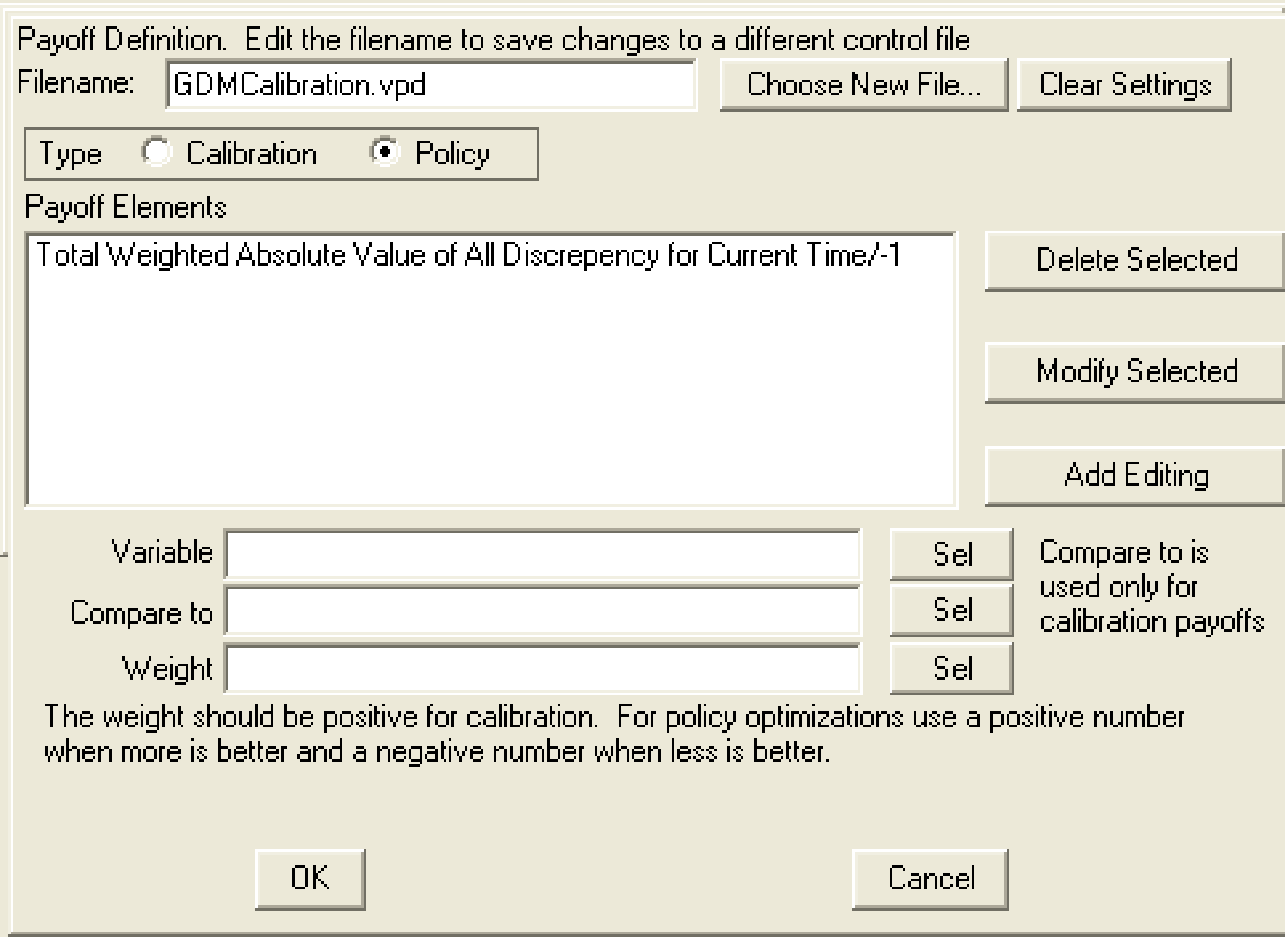
Running Calibrations in Vensim: (Under Model/Simulate Commands)



Optimization Control

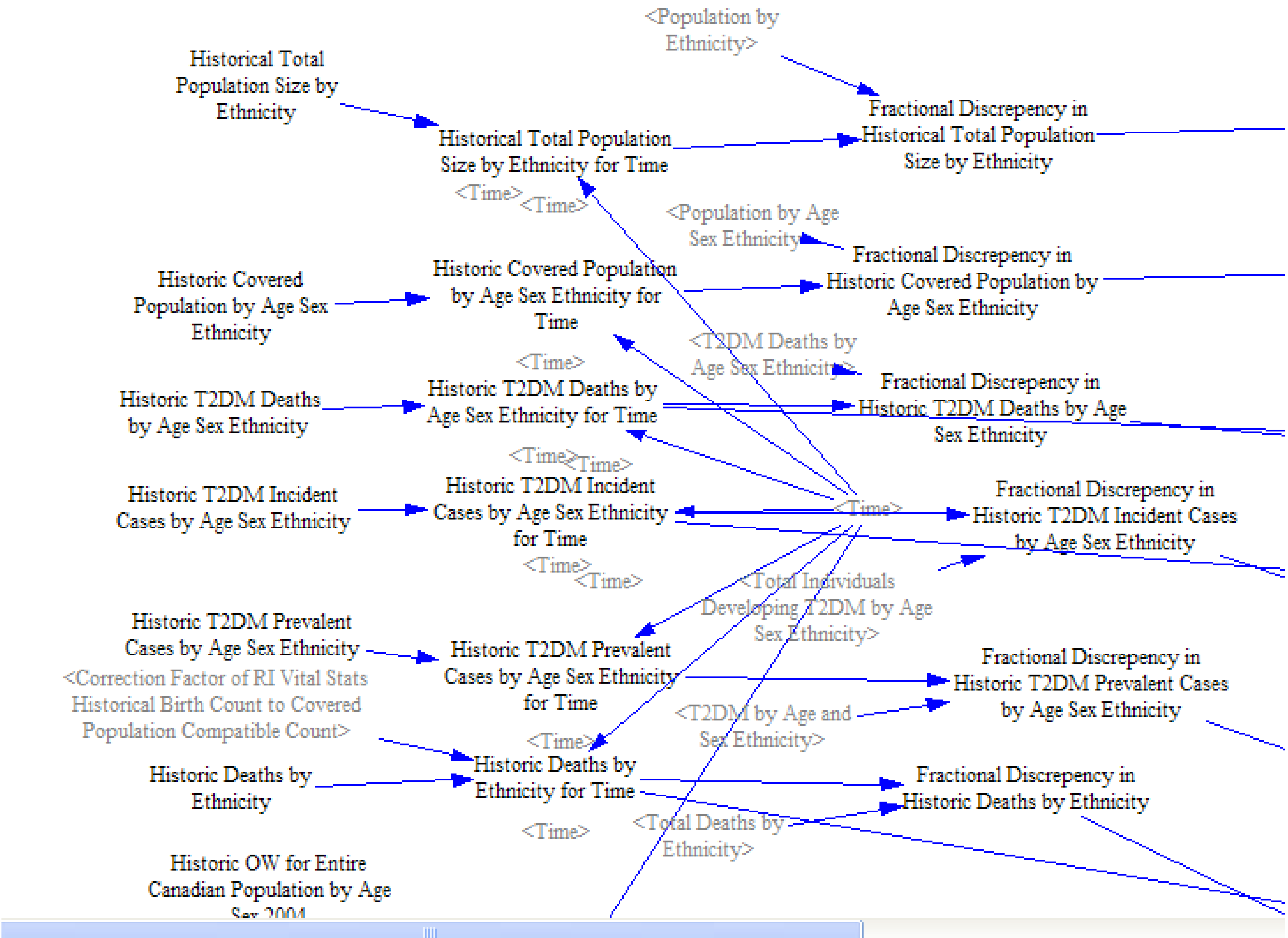


Payoff Definition



Assessing Model "Goodness of Fit"

- To improve the "goodness of fit" of the model to observed data, we need to provide some way of quantifying it!
- Within the model, we
  - For each historic data, calculate discrepency of model
    - Figure out absolute value of discrepency from comparing
      - Historic Data
      - The model's calculations
    - Convert the above to a fractional value (dividing by historic data)
  - Sum up these discrepency



<Population by Ethnicity>
Historical Total Population Size by Ethnicity
Historical Total Population Size by Ethnicity for Time
Fractional Discrepency in Historical Total Population Size by Ethnicity
<Time>
<Time>
<Population by Age Sex Ethnicity>
Historic Covered Population by Age Sex Ethnicity
Historic Covered Population by Age Sex Ethnicity for Time
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Historic T2DM Deaths by Age Sex Ethnicity
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<T2DM by Age and Sex Ethnicity>
<Time>
Historic Deaths by Ethnicity
Historic Deaths by Ethnicity for Time
Fractional Discrepency in Historic Deaths by Ethnicity
<Time>
<Total Deaths by Ethnicity>
Historic OW for Entire Canadian Population by Age Sex 2004

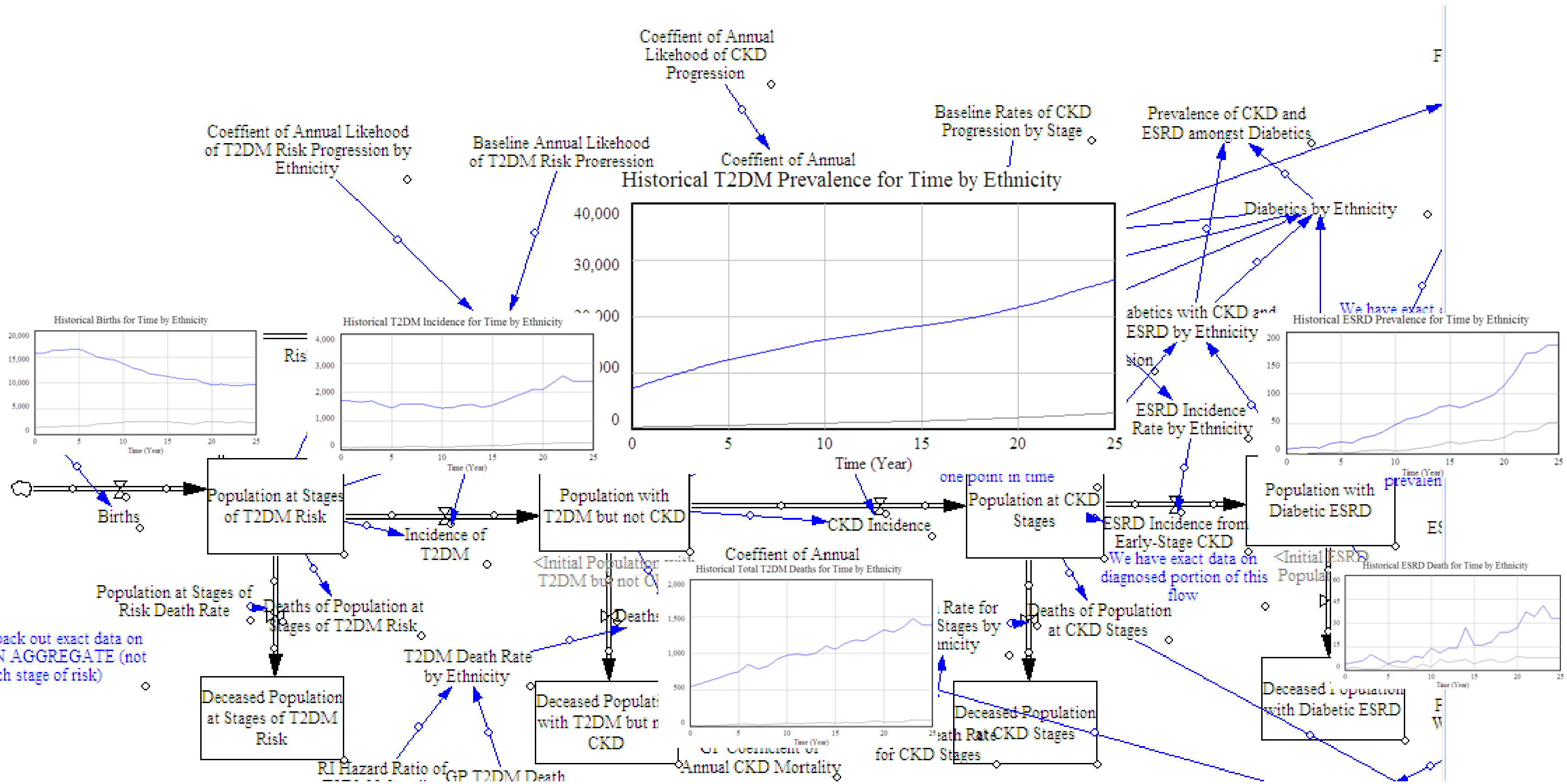
Required Information for Calibration

- Specification of what to match (and how much to care about each attempted match)
  - Involves an “error function” (“payoff function”, “penalty function”, “energy function”) that specifies “how far off we are” for a given run (how good the fit is)
- A statement of what parameters to vary, and over what range to vary them
- Characteristics of desired tuning algorithm
  - Single starting point of search?

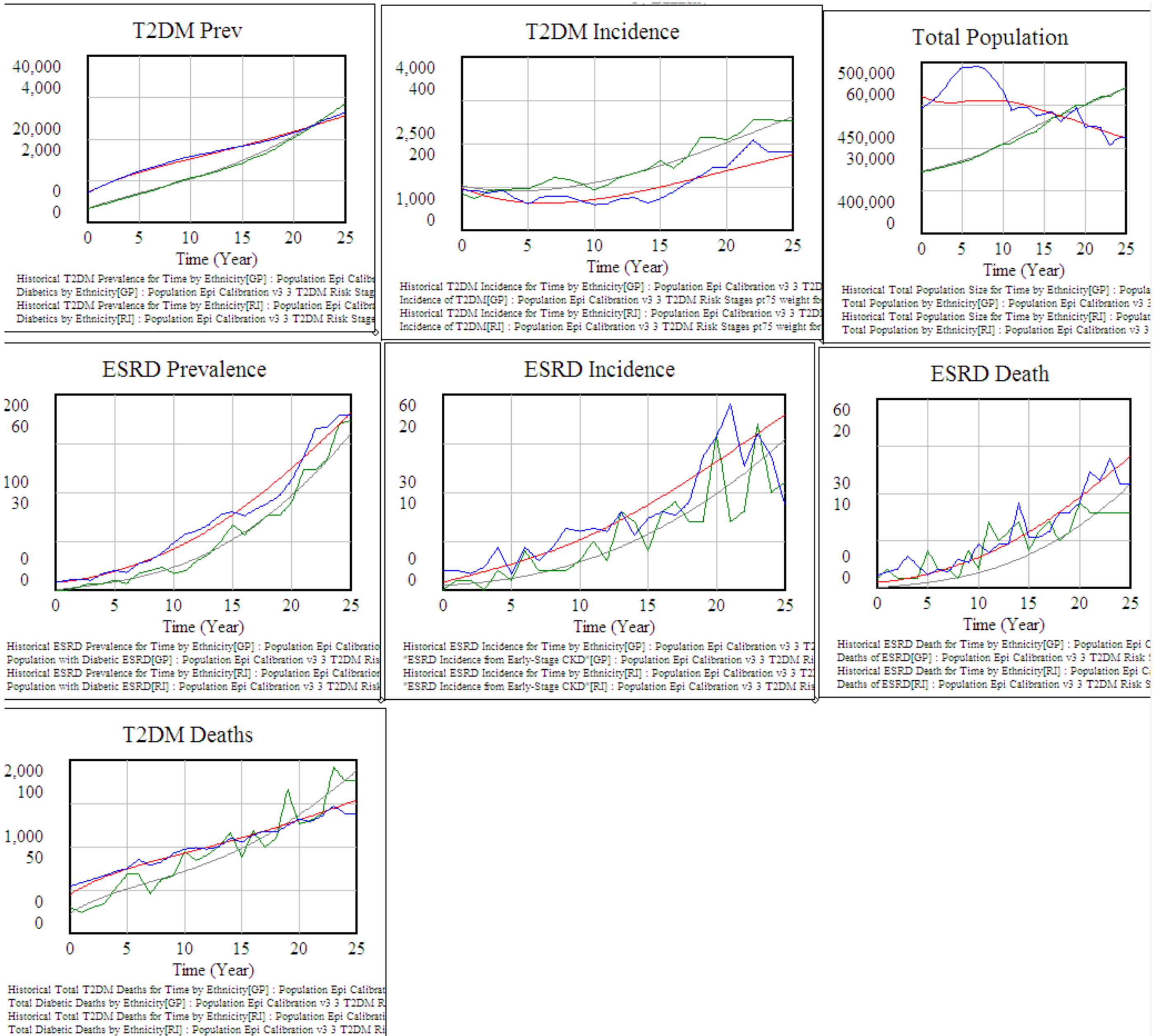
Considerations

- Adding constraints helps increase identifiability (selection of realistic best fit)
- Adding parameters to tune leads to larger space to explore
- Adding too many parameters to tune can lead to underdetermined situation
- All fits are within constraints of model

The Pieces of the Elephant
Example Model of Underlying Process & Time Series It Must Match



Single Model Matches Many Data Sources



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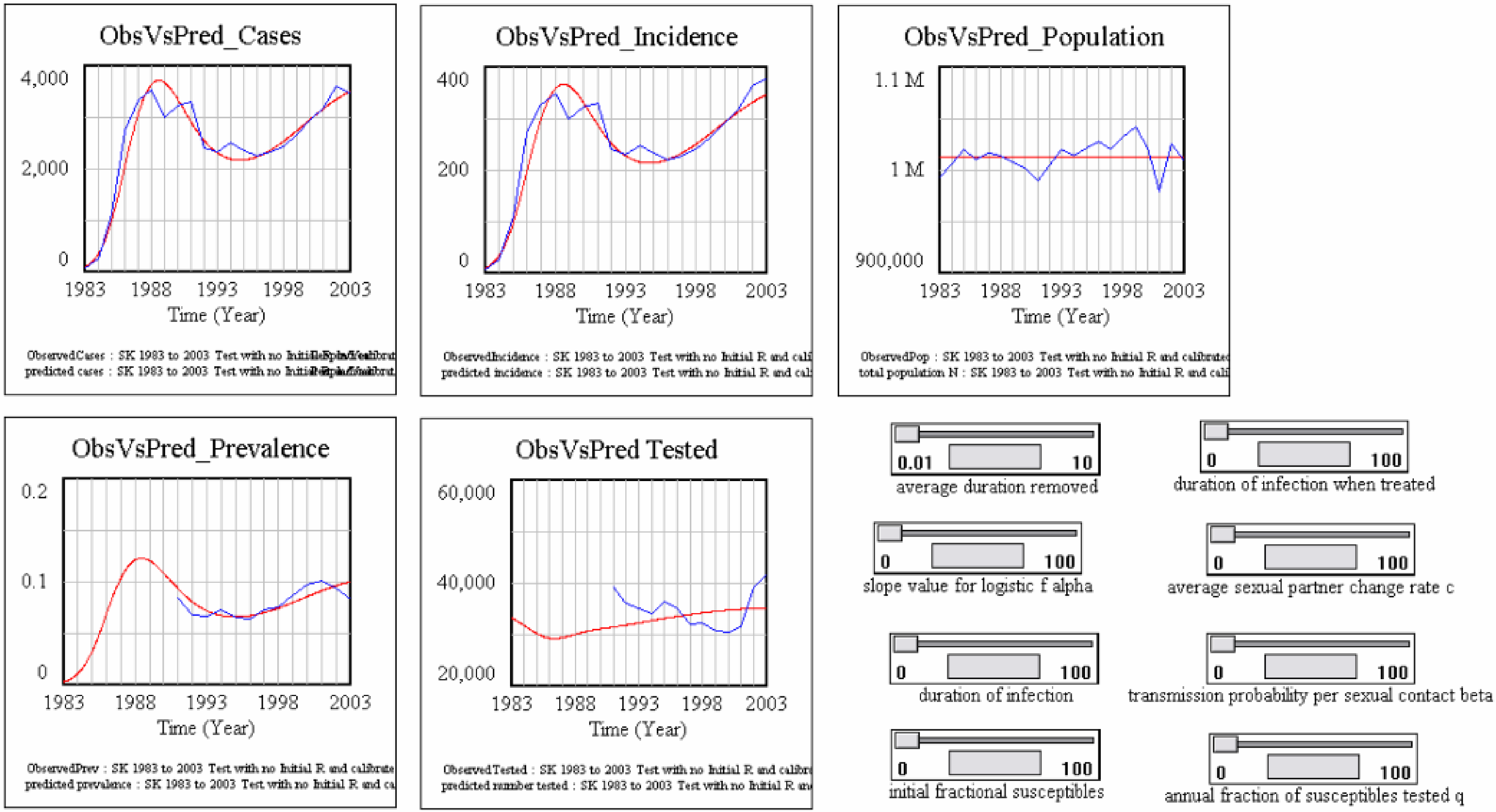
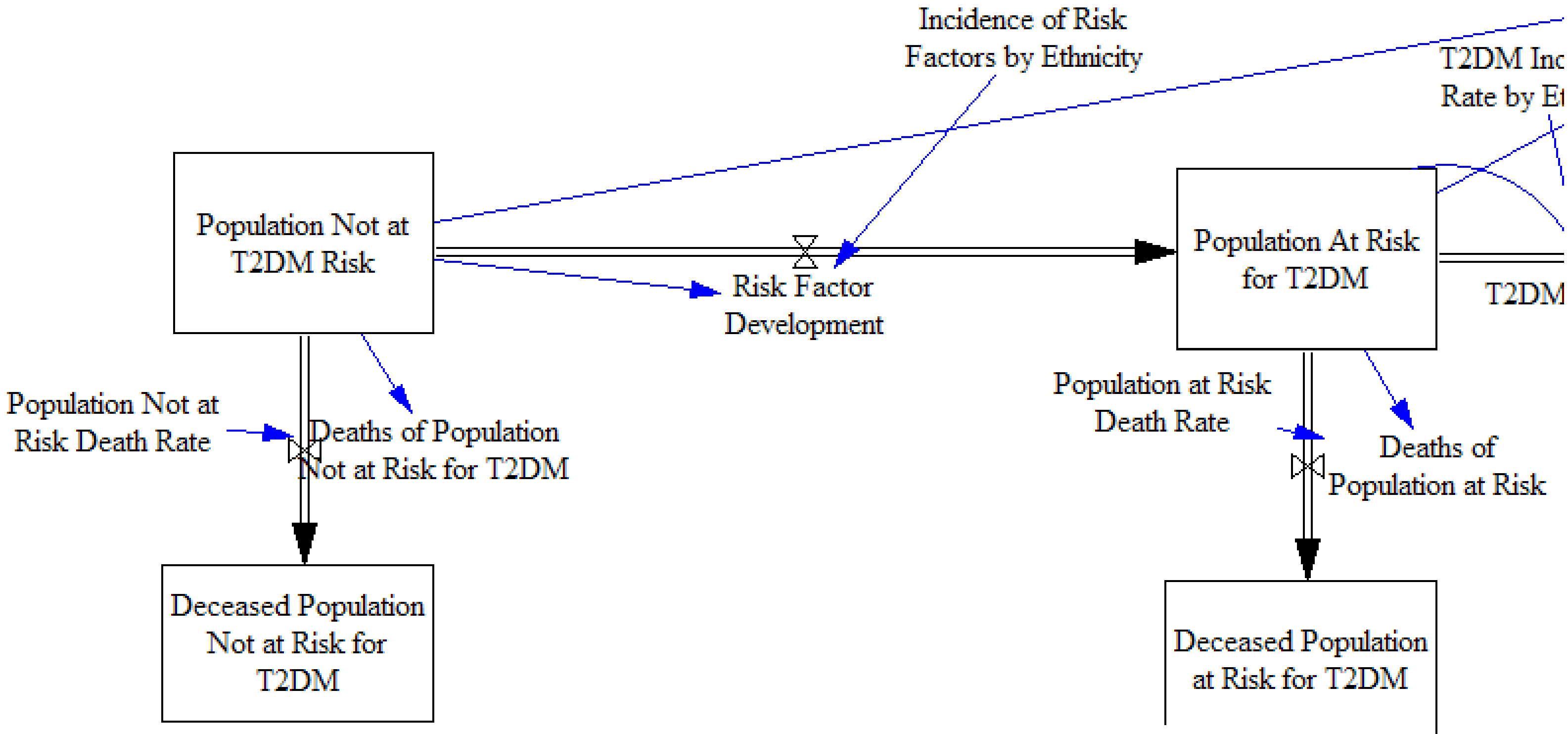
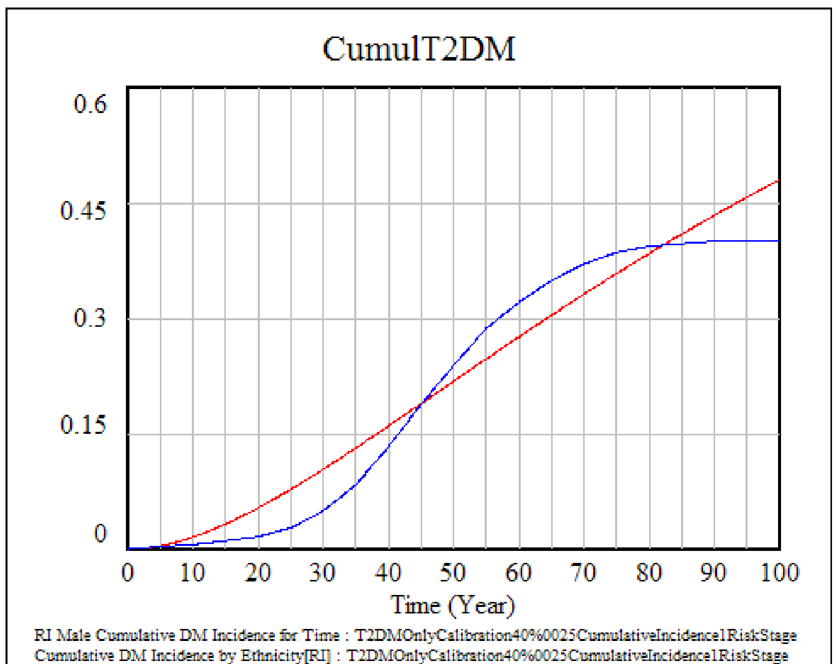
Pieces of the Elephant: STI



Science



Incidence of Risk Factors by Ethnicity
Population Not at T2DM Risk
Risk Factor Development
Population At Risk for T2DM
Population Not at Risk Death Rate
Deaths of Population Not at Risk for T2DM
Population at Risk Death Rate
Deaths of Population at Risk
Deceased Population Not at Risk for T2DM
Deceased Population at Risk for T2DM



CumulT2DM
0.6
0.45
0.3
0.15
0
0
10
20
30
40
50
60
70
80
90
100
Time (Year)
RI Male Cumulative DM Incidence for Time : T2DMOnlyCalibration40%0025CumulativeIncidence1RiskStage
Cumulative DM Incidence by Ethnicity[RI] : T2DMOnlyCalibration40%0025CumulativeIncidence1RiskStage